An Alternative Portfolio Theory

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PAST RESULTS ARE NOT NECESSARILY INDICATIVE OF FUTURE RESULTS

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Avoid Sharp(e) Drawdowns!

Introduction

In this note, we seek to optimize a portfolio allocation using the Serenity Ratio[1] developed in An Alternative Portfolio Theory (APT – Part 1). As a reminder, the Serenity Ratio considers path dependent variables to more accurately measure the drawdown risk of an investment strategy. The higher the Serenity, the better, thus an investor seeks to achieve the highest Serenity possible. However, in the context of a portfolio optimization, we find that the Serenity Ratio lacks predictability: it is a proven ex-post risk indicator to evaluate drawdowns but fails to provide information on potential future deeper drawdowns. Therefore, the Serenity Ratio cannot be used by investors to optimize their portfolio allocation. The second part of this paper focuses on finding a suitable indicator that shows the same properties as the Serenity Ratio (later called proxy) which could be used in a portfolio allocation and help limit the drawdown risk. To do so, we analyze the drivers of drawdowns and we find several explanatory variables, including autocorrelation of returns. We conclude by finding a good proxy that accounts for these variables which has a stronger predictive power than the Serenity Ratio while providing better results in minimizing drawdowns.

A Follow-Up on APT Part 1

In the last part of APT Part I, we proposed an alternative Risk-Return spectrum to the classical Markowitz (1952) Return-Volatility spectrum. We showed that our measure of Penalized Risk defined as Ulcer (average risk) multiplied by Penalty Factor (CDaR/Vol) was an excellent indicator of the hidden risks of drawdowns. In the first part of this paper, we extend our reasoning by defining optimal portfolios in both Markowitz (1952) and our alternative space. To be consistent with APT Part I, we use the same HFRI Indices[2] and define our portfolios over the same period (1990-2016), and we used a standard numerical optimization process as defined in Cheklov et al. (2005).

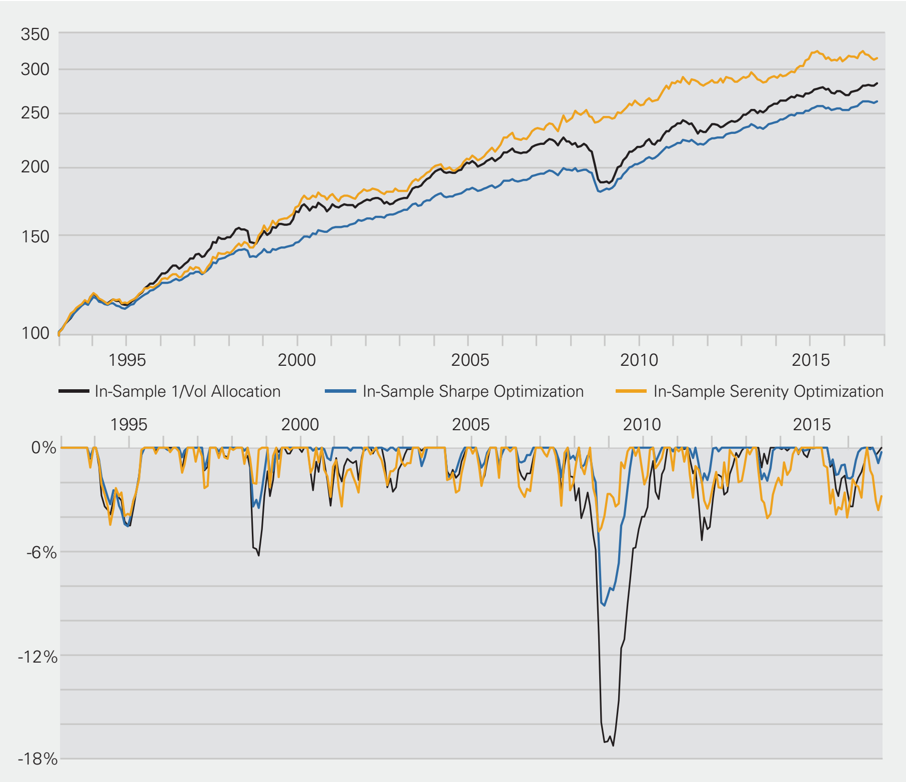
1 See APT Part 1 – Don't Get Trapped in a Pitfall, January 2017 - https://www.keyquant.com/Publications.

2 List and description is available in the appendices. Keep in mind that all the strategies are net of the risk-free rate.

1- In-Sample Sharpe and Serenity Ratios Optimizations

These in-sample optimizations represent the static allocation which would have maximized the Sharpe and Serenity ratios over the 1990-2016 time period, had the future been known in 1990. Figure 1 shows that the two resulting strategies would have yielded very different results. The classic risk parity allocation (1/vol) has been added to Figure 1 and Table 1 for comparison.

Figure 1 *NAVs of the In-Sample Sharpe and Serenity Optimization Processes*



Underwater Curves of the In-Sample Sharpe and Serenity Optimization Processes

As the in-sample Serenity optimization represents the very best static investment an investor could have made, we will use it thereafter as our benchmark. Our goal will be to find a practical allocation methodology that yields results as close as possible to this ideal optimum.

The optimal static allocations are shown on figure 2.

Figure 2 *Optimal In-Sample allocations*

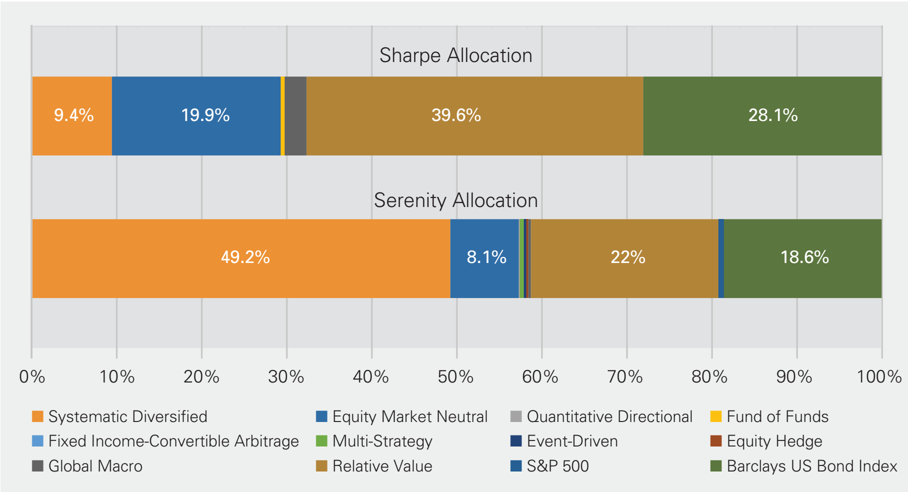


Table 1 *Statistics of the In-Sample Sharpe and Serenity Optimization Processes*

	In-Sample 1/Vol Allocation	In-Sample Sharpe Optimization	In-Sample Serenity Optimization
Return	4.43%	4.10%	4.89%
Volatility	4.02%	2.64%	4.16%
Sharpe Ratio	**1.10**	**1.56**	**1.18**
Ulcer Index	3.39%	1.68%	1.65%
UPI	**1.30**	**2.44**	**2.97**
Max DD	17.25%	9.11%	4.84%
CDaR	12.67%	6.29%	4.00%
Pitfall	3.15	2.38	0.96
Penalized Risk	10.7%	4.0%	1.6%
Serenity	**0.41**	**1.03**	**3.08**

A glossary with the definition of every risk metric used in the paper is available in the appendices. The results of each optimization are as follows:

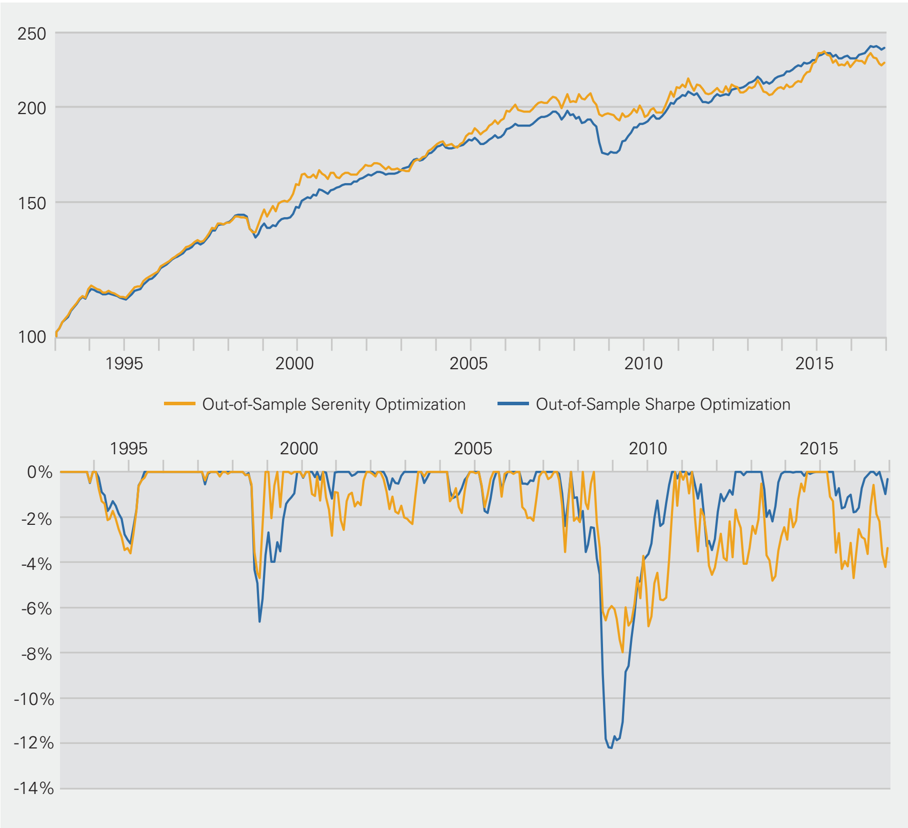
- The Sharpe optimization offers a low volatility strategy yet cannot avoid the 2008 financial crisis losses due to its investment into convergent strategies (Equity Market Neutral and Relative value) which suffer heavy drawdowns during that period.
- The Serenity optimization offers a higher volatility yet much lower drawdowns thanks to its high investment in divergent strategies (mainly Systematic Diversified).

Serenity Optimization sharply decreases the maximum drawdown

2- Out-of-Sample Sharpe and Serenity Ratios Optimizations

In order to test our indicator in the context of an actual portfolio optimization on a month by month basis, we now perform an out-of-sample optimization. Out-of-Sample optimization represents the rolling allocation which would have optimized the Sharpe and Serenity Ratios over the 1990-2016 period, rebalanced on a monthly basis. Figure 3 shows the result of the optimized portfolios.

Figure 3 *NAVs of the Out-of-Sample Sharpe and Serenity Optimization Processes*



Underwater Curves of the Out-of-Sample Sharpe and Serenity Optimization Processes

Table 2 *Statistics of the In-Sample (Static) and Out-of-Sample (Rolling) Sharpe and Serenity Optimizations*

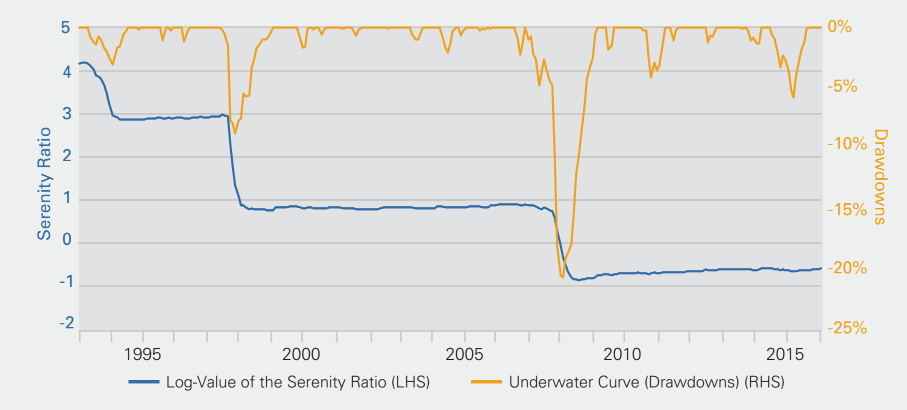
	Static Sharpe Optimization	Rolling Sharpe Optimization	Static Serenity Optimization	Rolling Serenity Optimization
Return		3.70%		3.50%
Volatility		2.88%		3.84%
Sharpe Ratio	**1.56**	**1.28**	**1.18**	**0.91**
		-18%		
Ulcer Index		2.56%		2.46%
UPI	**2.44**	**1.45**	**2.97**	**1.42**
Max DD		12.21%		8.00%
CDaR		9.63%		6.52%
Pitfall		3.35	**+95%**	1.70
Penalized Risk		8.6%		4.2%
Serenity	**1.03**	**0.43**	**3.08**	**0.84**
		-58%		**-72%**

When looking at both the Static and Rolling Sharpe Ratio optimizations, we see a decline of the resulting Sharpe Ratio of 18% (from 1.56 to 1.28) showing that Volatility is a quite stable and predictive measure of risk. However, the Serenity Ratio of both Sharpe optimizations drops by more than 50% (from 1.03 to 0.43) reminding us that Volatility is not the best estimator to predict drawdowns.

If we now look at the Static and Rolling Serenity optimizations, we see a 72% decline in the resulting Serenity Ratio (from 3.08 to 0.84) showing that despite being a good measure of drawdown risk, it is not predictive enough to be used in a practical portfolio optimization.

Indeed, the Serenity Ratio only provides relevant information to the investor if heavy drawdowns have been observed or when there is sufficient historical data to be representative of the behavior of the strategy through different market cycles.

Figure 4 *Serenity Ratio and Underwater Curve of the Relative Value Strategy*



The main issue with the Serenity Ratio is its instability and its dependency on extreme events. As an example, Figure 4 shows the value of the Serenity ratio and the underwater curve for the Relative Value Strategy. We see that each deep drawdown drastically reduces the value of the Serenity but does not provide information towards future potential deeper drawdowns.

Serenity Ratio tells you what happened, not what may happen

A Predictive Proxy For The Serenity Ratio

This part will focus on the origins of drawdowns in order to find a suitable alternative to the Serenity Ratio with stronger predictive properties that could be used in a portfolio optimization process.

1- In search of the lost autocorrelation

Following the methodology presented in Burghardt, Duncan and Liu (2003) by using Monte Carlo simulations of autoregressive processes (AR(1)) we showed that the risk of drawdowns (measured by Ulcer Index and Pitfall Indicator) is mainly impacted by three components[3]:

1- **Returns** ⟶ negative returns lead to higher drawdowns

2- **Volatility** ⟶ higher volatility can lead to higher drawdowns

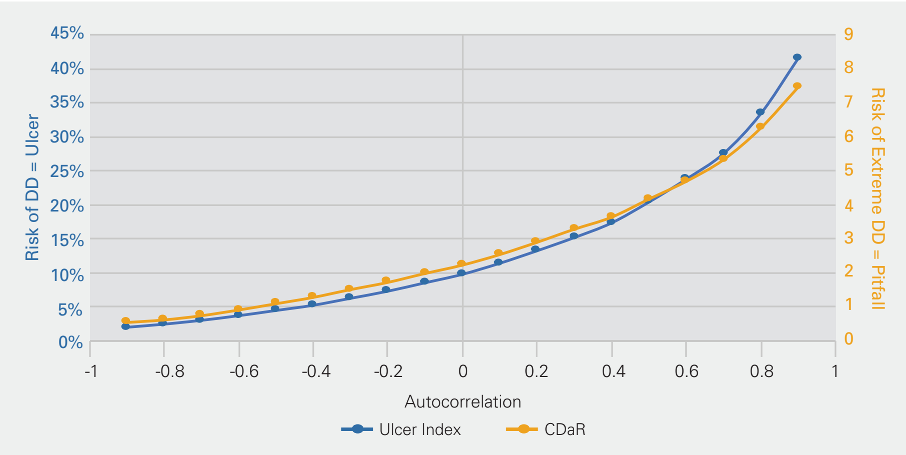
3- **Autocorrelation** ⟶ higher autocorrelation leads to a higher risk of drawdowns through the potential succession of negative returns

3 Burghardt G., Duncan R. and Liu L. (2003). Understanding Drawdowns. AlternativeEdge Research, Newedge Prime Brokerage, Working Paper.

While return and volatility are accounted for in the classic Sharpe approach, autocorrelation is ignored.

Figure 5 shows the impact of autocorrelation on the risk of drawdowns.[4] The exponential shape of both the Ulcer Index and the Pitfall Indicator curves show that higher levels of positive autocorrelations tend to produce very dangerous drawdowns that would result in an investment blow-up sooner or later.

Table 5 *Impact of the Autocorrelation on the risk of Drawdowns*



The same analysis of the impact of return and volatility is available in the appendices.

As a conclusion, a risk measure which takes into account returns, volatility and autocorrelation could be a good predictor of drawdowns and act as a proxy of the Serenity Ratio.

4 Based on simulations of AR(1) processes with a Mean of 5%, volatility of 10% and controlling for autocorrelation.

2- The Smart Sharpe Ratio

In The Statistics of Sharpe Ratio (Lo (2002)), it is shown that the actual time aggregation of the Volatility using the square root of time is a simplification of the real formula for annualizing the Volatility. The formula that is widely used by the industry to annualize the Volatility of an asset by multiplying the daily/monthly volatility by either $\sqrt{252}$ or $\sqrt{12}$ disregards the potential autocorrelation. The more accurate formula is[5]:

$$\sigma_n^2 = \sigma^2 \left(n + 2 \sum_{i=1}^{n} (n\text{-}i)\ \rho_i \right)$$

Where σ_n^2 is the annualized variance of a portfolio based on the variance (σ^2) calculated on n periods and the different autocorrelation coefficients ρ_i at lag i.

Should a process have all its autocorrelation coefficients equal to zero the formula becomes:

$$\sigma_n^2 = n\sigma^2$$

We find the widely-used formula for the annualization of volatility, which gives $Vol_{ann} = \sqrt{252}\ .Vol_{daily}$.

A parallel can be made between the Serenity formula and the accurate Sharpe formula:

$$\text{Serenity Ratio} = \frac{\text{Return}}{\text{Ulcer Index} \times \frac{\text{CDaR}}{\text{Vol}}} = \frac{\text{Return}}{\text{Ulcer Index} \times \text{Extreme Risk Penalty}}$$

$$\text{Sharpe Ratio} = \frac{\text{Return}}{\text{Vol} \times \sqrt{1 + 2\sum_{i=1}^{n} \frac{(n\text{-}i)}{n} \rho_i}} = \frac{\text{Return}}{\text{Vol} \times \text{Autocorrel. Penalty}}$$

Where both Volatility and Ulcer represent an average risk and $\frac{\text{CDaR}}{\text{Vol}}$ and $\sqrt{1 + 2\sum_{i=1}^{n} \frac{(n\text{-}i)}{n} \rho_i}$ represent a penalty factor affecting the average risk.

For the remainder of this paper, the standard Sharpe Ratio will be referred to as Traditional Sharpe Ratio while the accurate Sharpe Ratio taking autocorrelations into account will be referred as Smart Sharpe Ratio. The similarities between the Serenity Ratio and the Smart Sharpe make the latter a potential candidate as a proxy for future optimization. The next part will focus on verifying that the instability of Serenity Ratio (due to the instability of the Extreme Risk Penalty) is corrected in the Smart Sharpe.

The Smart Sharpe Ratio could act as a Proxy for the Serenity Ratio by penalizing autocorrelation

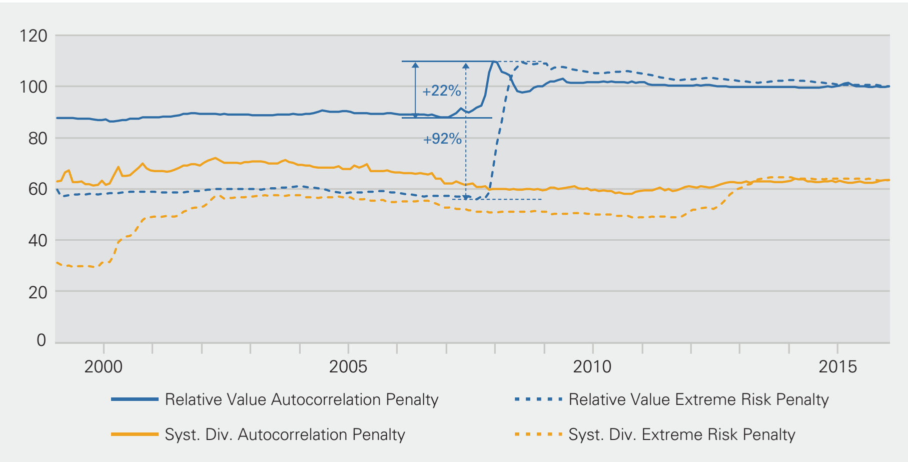
5 Demonstration in the appendices.

3- Stability of the Smart Sharpe Ratio

The main purpose of the Serenity Ratio was to penalize strategies with a hidden risk of drawdowns which could not be reflected in the sole value of the volatility. This feature has been transposed to the Smart Sharpe Ratio through the use of autocorrelation as shown previously (Fig. 5). We now need to verify the stability (hence the potential predictability) of the measure to make the Smart Sharpe Ratio a good proxy.

The stability issue in the Serenity Ratio relies mainly on the instability of the Extreme Risk Penalty. For example, the following graph shows the value of the Extreme Risk Penalty (CDaR/Vol) and the Autocorrelation Penalty ($\sqrt{1+2\sum_{i=1}^{n}\frac{(n\text{-}i)}{n}\rho_i}$) for the Relative Value and the Systematic Diversified Strategies[6]:

Figure 6 *Autocorrelation Penalty and Extreme Risk Penalty*



We see that the 2008 financial crisis has a very significant impact on the value of the Extreme Risk Penalty of the Relative Value (92% growth from 56.9 to 109), showing once again the dependency of Serenity (through CDaR) to extreme events. In the meantime, the Autocorrelation Penalty only moves by 22% (from 90 to 110) showing that our new Penalty Factor is a more stable value. This is also true when looking at the stability of the Autocorrelation Penalty of the Systematic Diversified strategy. The same analysis on other strategies is available in the appendices.

6 We chose to rebase both Penalty Factors of the Relative Value Strategy to 100 at the end of 2016 for comparison.

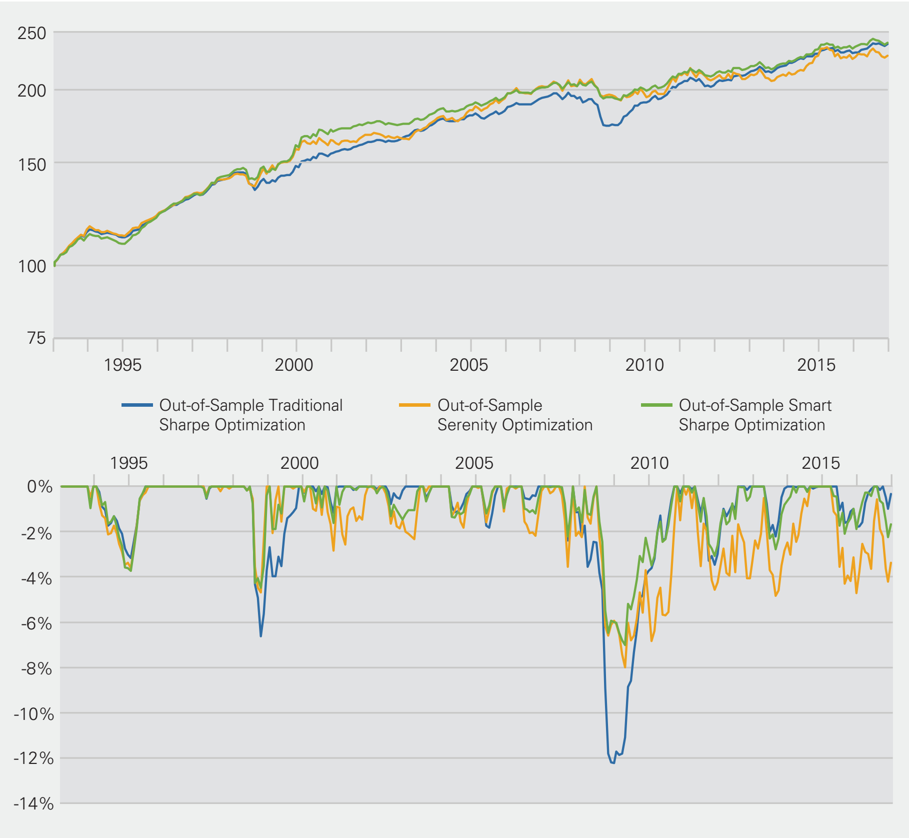
Therefore, the Smart Sharpe Ratio shares the main properties of the Serenity Ratio regarding drawdowns and its Penalty Factor is more stable than the Serenity Ratio which makes it a very suitable proxy to test in an optimization process to confirm its superior predictive power. The Serenity Ratio will however remain our risk measure of choice to compare ex-post strategies.

Smart Sharpe Ratio is a good proxy of the Serenity Ratio

4- Smart Sharpe Ratio Out-of-Sample Optimization

The result of out-of-sample optimization confirms that the Smart Sharpe Ratio can be used in a portfolio allocation to improve the drawdown profile of an investment and therefore its Serenity Ratio. The following graphs present the NAVs and the underwater curves of the Traditional Sharpe, the Serenity and the Smart Sharpe strategies:

Figure 7 *NAVs of the Traditional Sharpe, the Serenity and the Smart Sharpe Optimization*



Underwater Curves of the Traditional Sharpe, the Serenity and the Smart Sharpe Optimization

The Smart Sharpe Optimization confirms its ability to anticipate drawdowns with the use of auto-correlations compared to the Traditional Sharpe Optimization. The strategy also offers the best Serenity Ratio of all optimizations by having fewer drawdowns (lower Ulcer) and a very good CDaR as shown in the following statistics.

Table 3 *Statistics of the Out-of-Sample Traditional Sharpe, Serenity and Smart Sharpe Optimization Processes*

	Traditional Sharpe Optimization	Serenity Ratio Optimization	Smart Sharpe Ratio Optimization
Return	3.70%	3.50%	3.72%
Volatility	2.88%	3.84%	3.06%
Traditional. Sharpe	**1.28**	**0.91**	**1.22**
Smart Sharpe	**0.97**	**0.91**	**1.12**
		≈	
Ulcer Index	2.56%	2.46%	1.68%
UPI	**1.45**	**1.42**	**2.21**
Max DD	12.2%	8.0%	7.0%
CDaR	9.63%	6.52%	5.61%
Pitfall	3.35	1.70	1.84
Pen Risk	8.6%	4.2%	3.1%
Serenity	**0.43**	**0.84**	**1.21**
	x 2		x 1.5

Table 3 shows the improvements we progressively made starting from a Traditional Sharpe optimization to a Smart Sharpe Optimization. Starting from a Serenity Ratio of 0.43, using the autocorrelation as a proxy for the risk of drawdowns, we were able to increase the Serenity up to 1.21. Moreover, the Sharpe Ratio of the Smart Sharpe Optimization shows that using autocorrelations in an allocation process does not trade off the risk of volatility for a risk of drawdowns but tends to minimize both.

The Smart Sharpe is good at managing the average risk of drawdowns (Ulcer Index of the out-of-sample Smart Sharpe Optimization is 1.68%, very close to its in-sample-value of 1.65%, cf. Table 1). However, it is less efficient in reducing unexpected drawdowns as they are not as well measured through autocorrelations which can represent a possible improvement of the measure.

The superior predictive power of the Smart Sharpe Ratio offers much better drawdown control while preserving the Sharpe Ratio

A Practical use of the Smart Sharpe Ratio Optimization

1- Results of a constrained out-of-sample optimization

The previous optimization did not consider problems and constraints many allocators may face. In order to be closer to these requirements, we fixed the following constraints when calculating the optimal portfolio[7]:

- All metrics are calculated with a 98% exponential smoothing average
- Allocation in each asset is capped at 20% maximum
- Weights can only move by 4% each month

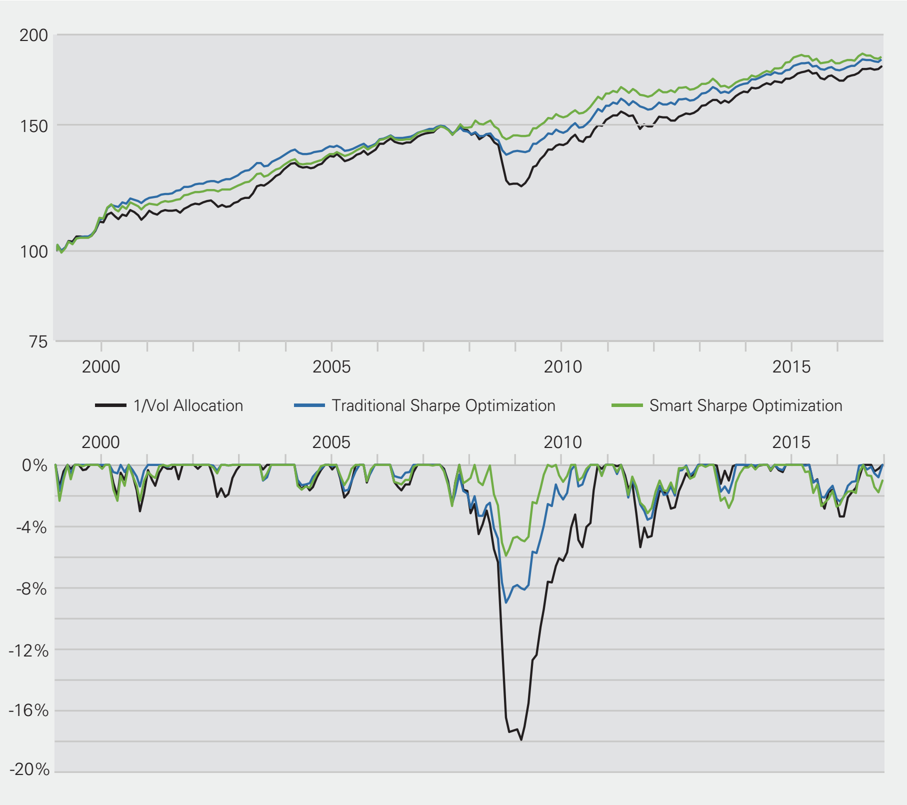
We decided not to constrain any investment in any particular asset class (such as Equity or Bonds) even though many investors may face these requirements. The main purpose of this optimisation is to understand how these asset classes can compose a better portfolio using the Smart Sharpe Ratio.

The results of the optimizations over the 1999-2016 period[8] are presented on Figure 8 and Table 4 (see below).

7 Robustness checks on the parameters have been performed to ensure the consistency of the results.

8 Here we dropped the 1990-1999 period because it does not represent a differentiation period for the strategies.

Figure 8 *NAVs of the Traditional Sharpe and Smart Sharpe Optimization Processes*



Underwater Curves of the Traditional Sharpe and Smart Sharpe Optimization Processes

Table 4 *Statistics of the Out-of-Sample Traditional Sharpe, Serenity and Smart Sharpe Optimization*

	1/Vol	Traditional Sharpe Optimization		Smart Sharpe Optimization
Return	3.34%	3.47%		3.53%
Vol	3.87%	2.88%		3.15%
Traditional Sharpe	**0.86**	**1.20**		**1.12**
Smart Sharpe	**0.67**	**1.11**		**1.14**
Ulcer	4.02%	2.05%		1.43%
UPI	**0.83**	**1.69**		**2.47**
Max DD	17.88%	8.94%		5.90%
CDaR	15.53 %	7.62%		4.63%
Pitfall Indicator	4.01	2.64		1.47
Penalized Risk	16.1%	5.4%	x 2.6	2.1%
Serenity	**0.21**	**0.64**	→	**1.68**

Figure 8 and Table 4 show the importance of taking autocorrelations into account when optimizing a portfolio. The drawdown control offered by the Smart Sharpe Portfolio is better than the Traditional Sharpe Portfolio while keeping similar Sharpe Ratios. The average risk of drawdown is vastly reduced with an Ulcer Performance Index (UPI) of 2.47 for the Smart Sharpe Optimization versus a UPI of 1.69 for the Traditional Sharpe Portfolio. The global risk of the strategy is also reduced as illustrated in the value of the Serenity Ratio of the Smart Sharpe strategy which is more than two times the value of the Serenity of the Traditional Sharpe strategy (1.68 vs. 0.64).

Are Investors Truly Using Sharpe Ratio?!

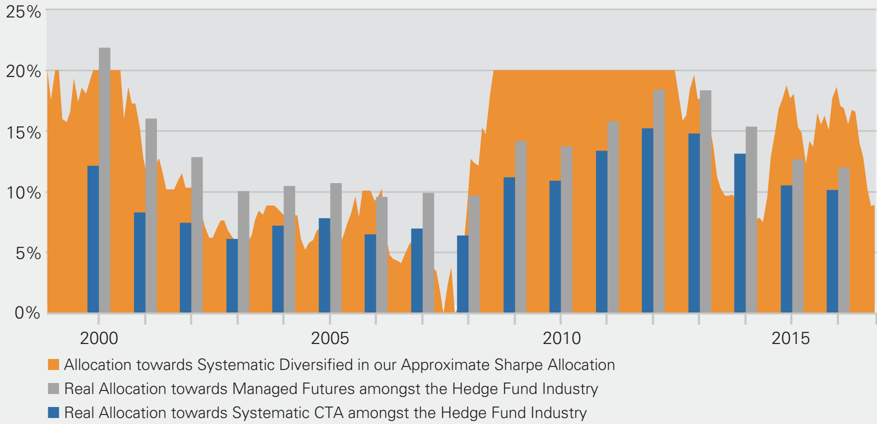


Figure 9 *Allocation in Systematic Diversified (CTA), Barclays Hedge*

We have compared the allocation to Systematic Strategies actually made by investors with our previous Traditional Sharpe allocation.

The divestment in Systematic Diversified Strategies after major financial turmoil, as seen between 1999 and 2008, and since 2013 is reflected in the actual allocation investors made towards CTAs.

2- Portfolio Allocations

Figure 10 *Optimal Allocation with the Traditional Sharpe*

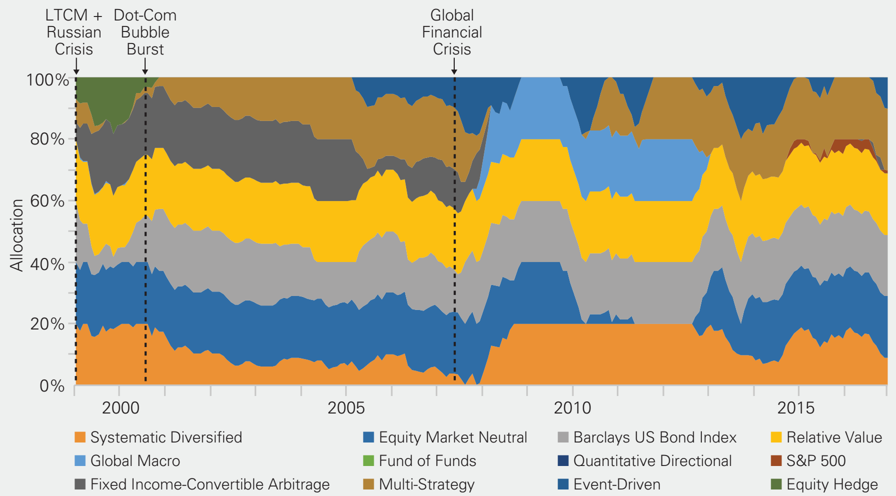
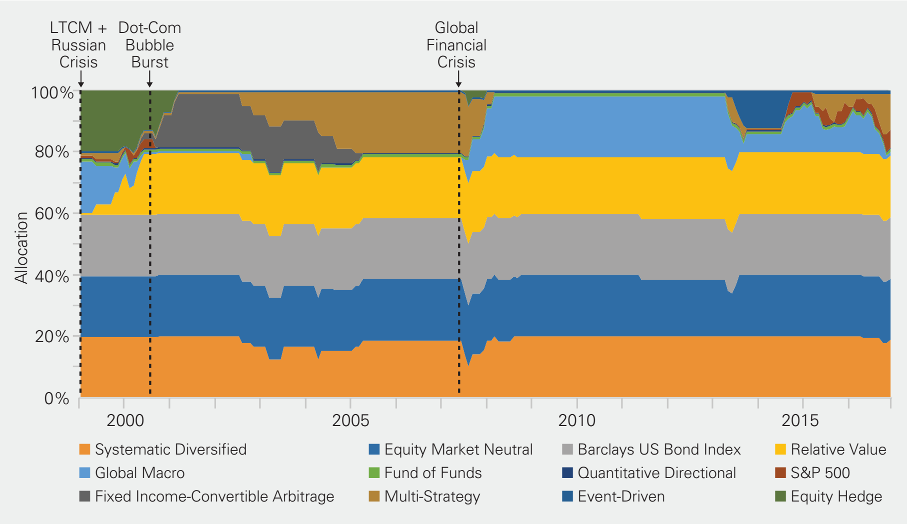


Figure 10 shows the weighting evolution of the portfolio allocation based on the Traditional Sharpe Ratio using the constraints defined previously. Using the Traditional Sharpe Ratio an investor won't have a very stable allocation through time. Following the Russian and LTCM crisis that occurred at the end of 1998, the fear of a more global propagation to the economy leads to a portfolio with a maxed-out allocation towards crisis alpha strategies (20% in Systematic Diversified). As the Russian crisis fades away and the dot-com bubble slowly deflates the investment in crisis alpha strategies is progressively reduced. The reduction of Systematic Diversified to the benefit of strategies with lower volatility but a higher risk of drawdowns is typical of the Traditional Sharpe Ratio optimization. The portfolio looks for higher returns and lower volatility even though the risk of potential future crisis still exists. The position in Systematic Diversified is only progressively rebuilt as the 2008 crisis unfolds and reaches a maximum just before the market turning point, therefore not providing enough crisis alpha during the heat of the market downturn. By doing this, investors can improve their short-term performance but these small gains risk being wiped out when the next downturn occurs. By being always one beat late regarding their investments, investors who reduce their exposure to CTAs trade small gains in an already positive environment for more risk of losing much more when things go sour.

Figure 11 shows the weighting evolution of portfolio allocation based on the Smart Sharpe Ratio using the constraints defined previously.

Figure 11 *Optimal Allocation using the Smart Sharpe Optimization*



The superior results and the over performance (Serenity Ratio of 1.68 vs. 0.64) of the Smart Sharpe Ratio optimization is reflected in its allocation over the 1999-2016 period. We can see an almost static allocation towards four complementary asset classes that constitute a core portfolio: Systematic Diversified, Equity Market Neutral, Relative Value and Bonds. The remaining 20% of the allocation evolves around allocating towards different strategies depending on the market conditions. The Smart Sharpe allocates towards another crisis alpha strategy (Global Macro) during the 2008 financial crisis and is able to invest in strategies with better return potential during market expansions. The difference with the Traditional Sharpe Strategy is that the investment in Systematic diversified stays constant during the whole period and unleashes its full crisis alpha potential as the 2008 downturn happens. By following the Smart Sharpe Ratio allocation, an investor might be trading off some potentially lower short-term performance for a more stable long-term growth and therefore live with less perceived and real risk regarding its investment.

Smart Sharpe Ratio is able to allocate efficiently towards complementary (divergent and convergent) strategies to build a portfolio that limits drawdowns, preserves the benefit of Sharpe Ratio while providing a better Serenity Ratio.

Conclusion

Following the Alternative Portfolio – Part 1, the purpose of this paper was to allocate a portfolio in order to maximize the Serenity Ratio. Because of its latency issues, Serenity Ratio cannot be used to optimize a portfolio, but only as an ex-post measure. Through the use of autocorrelations, we were able to find a proxy - The Smart Sharpe Ratio - that shares the properties of the Serenity Ratio and has better predictive properties. The resulting portfolio shows a greater stability in its allocation, limits the drawdown risk and provides a better Serenity Ratio while preserving the Sharpe Ratio. Therefore, investors who want to minimize their drawdown, should aim to maximize their Serenity Ratio by using the Smart Sharpe optimization.

References and Appendices

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Appendices – Part II

1- Glossary

Metric	Definition	Extra Info
Ulcer Index	Root Mean Square of Drawdowns	Measure of the average Risk of Drawdowns (the lower the better)
UPI (Ulcer Performance Index)	$UPI = \frac{\text{Return}}{\text{Ulcer Index}}$	"Sharpe Ratio"-like Indicator (Return over Average Risk of Drawdowns) (the higher the better)
CDaR(95%)	Average of the 5% "biggest" drawdowns	Measure of the Extreme Risk of Drawdowns (the lower the better)
Pitfall Ind.	$\frac{\text{CDaR(95\%)}}{\text{Vol}}$	Penalty Factor - Measure of the Extreme Risk of Drawdowns in number of volatilities (the lower the better)
Penalized Risk	Ulcer x Pitfall	Measure of the Global Risk of Drawdowns (lower is better)
Serenity	$\frac{\text{Return}}{\text{Pen. Risk}}$	"Sharpe Ratio"-like Indicator (Return over Global Risk of Drawdowns) (the higher the better)

2- HFRI Indices

HFRI Names	Short Name	Strategies Included
HFRI Macro: Systematic Diversified Index (HFRIMTI Index)	Systematic Diversified	Managed Futures, Trend Following
HFRI EH: Equity Market Neutral Index (HFRIEMNI Index)	Equity Market Neutral	Quantitaztive Equity Market Neutral Strategies
HFRI EH: Quantitative Directional (HFRIENHI Index)	Quantitative Directional	Factor-Based and Statistical Arbitrage Trading Strategies
HFRI FOF: Diversified Index (HFRIFOFD Index)	Fund of Funds	Investment in a variety of strategies among multiple managers
HFRI RV: Fixed Income-Convertible Arbitrage Index (HFRICAI Index)	Fixed Income-Convertible Arbitrage	Relative Value Strategies limited to Fixed Income and Convertible Instruments
HFRI RV: Multi-Strategy Index (HFRIFI Index)	Multi-Strategy	Relative Value Strategies on Fixed Income, derivatives, Equity, Real Estate and/or MLP Assets
HFRI Event-Driven (Total) Index (HFRIEDI Index)	Event-Driven	Event Driven Strategies
HFRI Equity Hedge (Total) Index (HFRIEHI Index)	Equity Hedge	Long-Short Equity Strategies
HFRI Macro (Total) Index (HFRIMI Index)	Global Macro	Global Macro Strategies
HFRI Relative Value (Total) Index (HFRIRVA Index)	Relative Value	Relative Value Strategies
S&P 500 (SPXT Index)	S&P 500	S&P 500
Barclays US Bond Index (LBUSTRUU Index)	Barclays US Bond Index	US Bonds

3- Smart Sharpe Ratio Demonstration

In this annex, we show that:

$$\sigma_n^2=\sigma^2\left(n+2\sum_{i=1}^{n}(n\text{-}i)\ \rho_i\right)$$

Considering the case of IID returns, we note $R_t(n)$ the returns over n periods:

$$R_t(n)=R_t+R_{t\text{-}1}+\ldots+R_{t\text{-}n+1}$$

We have:

$$\mathrm{Var}\big(R_t(n)\big)=\mathrm{Var}(R_t+R_{t\text{-}1}+\ldots+R_{t\text{-}n+1})=\sum_{i=0}^{n\text{-}1}\sum_{j=0}^{n\text{-}1}\mathrm{Cov}(R_{t\text{-}i},R_{t\text{-}j})$$

This can be represented as the sum of the coefficients of the following matrix of size n:

$$\begin{bmatrix} \mathrm{Cov}(R_t,R_t) & \cdots & \mathrm{Cov}(R_t,R_{t\text{-}n+1}) \\ \vdots & \ddots & \vdots \\ \mathrm{Cov}(R_{t\text{-}n+1},R_t) & \cdots & \mathrm{Cov}(R_{t\text{-}n+1},R_{t\text{-}n+1}) \end{bmatrix}$$

The matrix being symmetric, we split the sum in two parts, the sum of the diagonal coefficients and two times the sum of the upper triangle:

$$\mathrm{Var}\big(R_t(n)\big)=\sum_{i=0}^{n\text{-}1}\mathrm{Cov}(R_{t\text{-}i},R_{t\text{-}i})+2\sum_{i<j}^{n\text{-}1}\mathrm{Cov}\big(R_{t\text{-}i},R_{t\text{-}j}\big)$$

For the second part of the Sum we see that by adding through the consecutive diagonals we have:

$$\mathrm{Var}\big(R_t(n)\big)=\sum_{i=0}^{n\text{-}1}\mathrm{Cov}(R_{t\text{-}i},R_{t\text{-}i})+2\sum_{i=0}^{n\text{-}1}\sum_{k=1}^{n\text{-}1}(n\text{-}i)\mathrm{Cov}(R_{t\text{-}i},R_{t\text{-}i\text{-}k})$$

Then:

$$\mathrm{Var}\big(R_t(n)\big)=n\sigma^2+2\sum_{i=1}^{n\text{-}1}(n\text{-}i)\rho_i\sigma^2$$

4- AR(1) Filter

Due to the monthly granularity of our data set and the fact that the Smart Sharpe Ratio needs high lags of autocorrelations to be calculated, we have decided to apply an AR(1) filter on autocorrelations to limit the impact of estimation errors.

An AR(1) process is defined as follows:

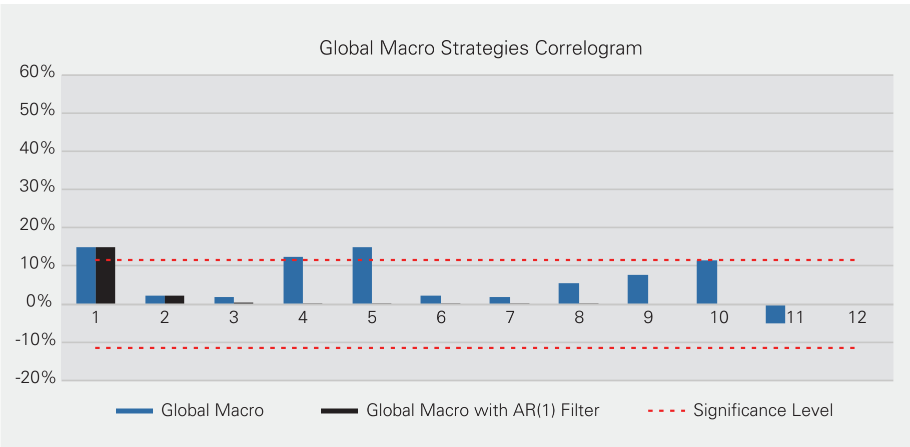
$$X_t = \Phi X_{t-1} + \epsilon$$

Where ϵ is a white noise.

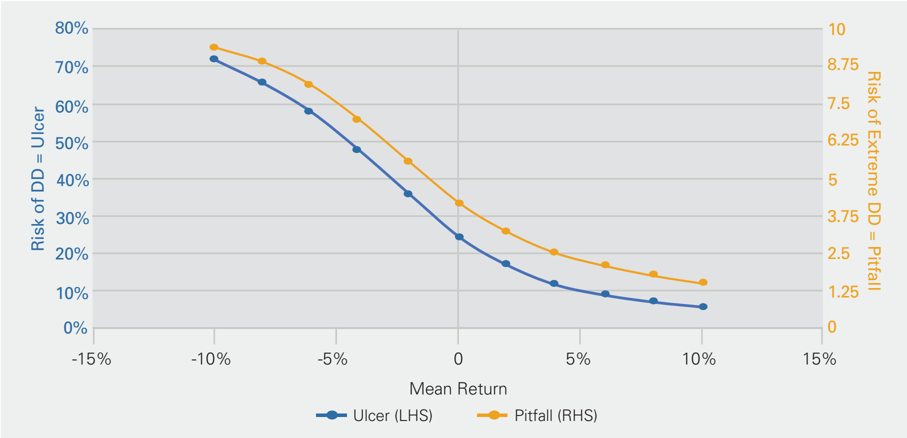
We can show that $\Phi = \rho_1 = \rho = \mathrm{Cor}(X_t, X_{t-1})$ and $\forall i > 0 \ \rho_i = \rho^i$.

Using this formula, we see that the calculation of the Smart Sharpe Ratio is made easier by only estimating the first autocorrelation coefficient and deriving the following coefficient from the first. This method prevents side effects in the calculation such as "negative" volatilities, non-exponentially decreases in the correlograms, etc.

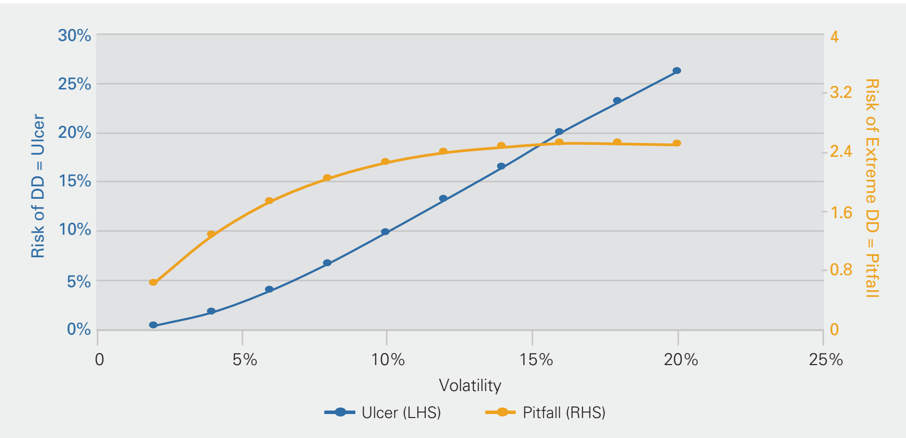
An example with the global Macro strategies showing estimation errors on high lags autocorrelation causing side effects on the calculation of the Smart Sharpe Ratio is shown below:



5- Impact of Return and Volatility on the Risk of Drawdowns



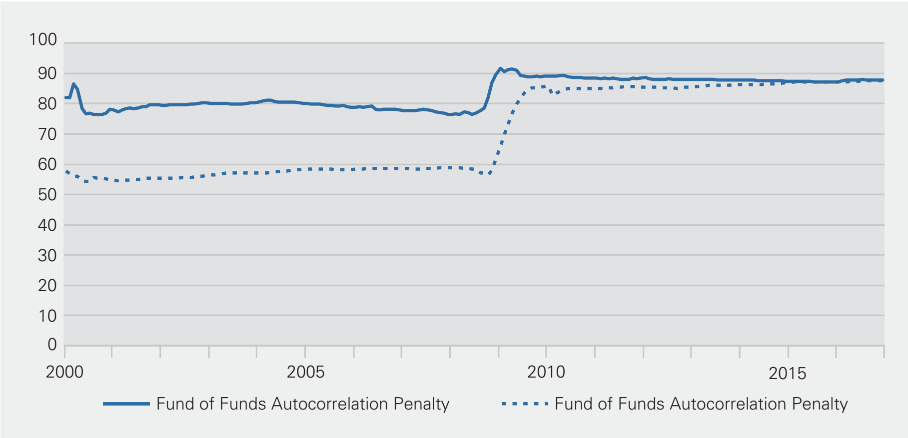
A strategy with negative returns would more likely suffer bigger drawdowns than a strategy with positive returns. This is confirmed by the simulations of an AR(1) process where the autocorrelation has been set to 0 and volatility to 10% controlling for returns between -10% and 10%, the risk of drawdowns (DD) diminishes as returns gets higher.

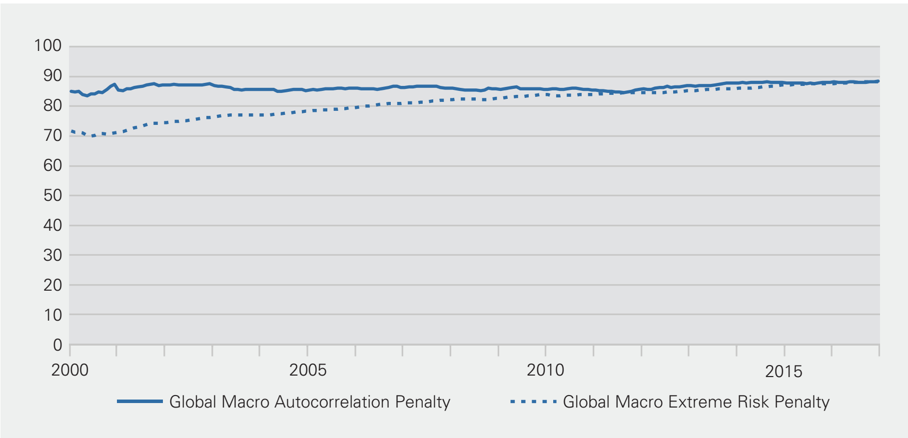


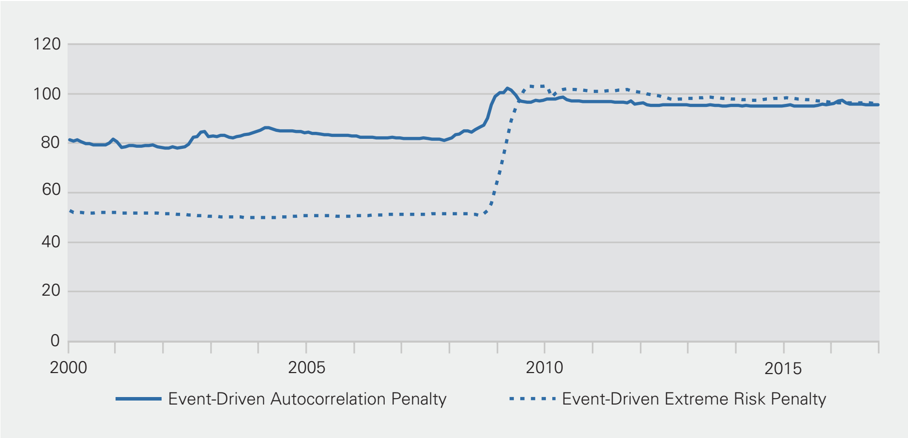
Higher volatility leads to higher average and extreme drawdowns as both Ulcer and Pitfall increase with volatility. The convergence of the Pitfall Indicator is due to the fact that when volatility is high compared to the returns, drawdowns of more than 95% occur therefore capping the value of CDaR to a maximum.

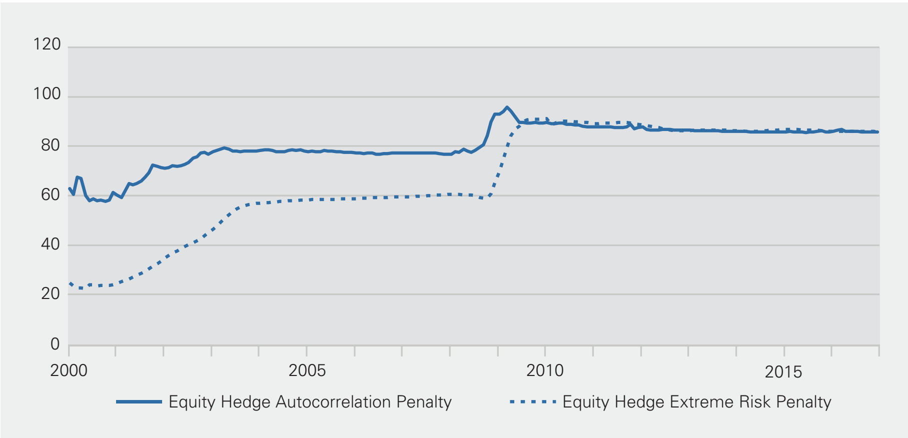
6- Penalty Factors

The following graphs represent the stability of the Autocorrelation Penalty compared to the Extreme Risk Penalty for other Strategies than those presented in Figure 6.






120
100
80
60
40
20
0
2000
2005
2010
2015
Event-Driven Autocorrelation Penalty
Event-Driven Extreme Risk Penalty


120
100
80
60
40
20
0
2000
2005
2010
2015
Equity Hedge Autocorrelation Penalty
Equity Hedge Extreme Risk Penalty

Notes

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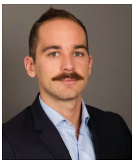

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