



An Alternative Portfolio Theory

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Part I: Serenity Ratio: Don't get Trapped in a Pitfall!



Don't get Trapped in a Pitfall!

Modern Portfolio Theory and its Discontents

In this note, we argue that classic risk measures (such as volatility) do not consider the path followed by returns and leave out the risk of potential large drawdowns. We also show that using path dependent measures such as the Ulcer Index or Conditional Drawdown at Risk adds valuable information to any investment decision. We propose a new measure of the global risk: The Serenity Ratio which reveals both the average and extreme risk carried by an investment. We conclude that this indicator can be used in a modified version of the Modern Portfolio Theory [Markowitz (1952)] and help investors choose the best strategies for their portfolio.

The Flaws of Classic Risk Measures

Annualized standard deviation or volatility (Sharpe Ratio being its return adjusted version) is one of the most widely used measures of the riskiness of an investment. However, this measure presents some drawbacks of which investors may not be aware. Both upside and downside changes in prices are used to calculate the volatility of an investment.

Many investors therefore use a modified version of standard deviation that only penalizes the downside risk: Downside Volatility (Sortino Ratio [Rom (1983)] being its return adjusted version). This downside risk measure can be misleading¹ when it comes to fat tail distributions.

The Omega Ratio [Shadwick and Keating (2002b)] was developed to overcome these nonnormal limitations by taking into account all moments of the distribution function.

However, all these measures do not consider autocorrelation of returns (i.e., today's return is dependent on yesterday's return).

Consider the three following strategies^{2, 3}:

- An investment in the MSCI World Index ("Long" Strategy)
- An investment in an asset where we rearranged the returns of the MSCI World Index so that the worst months occur first, ahead of the best months ("Sorted" Strategy)
- An investment in an asset where we rearranged the returns of the MSCI World Index in order to minimize drawdowns ("Flattened" Strategy)

¹ The measure does not focus on tail loss and therefore may significantly understate the range of potential losses. 2 The ideas presented in this paper are adapted from Martin (1987).

³ In this article, we always subtract the risk free rate (LIBOR) from our noted strategies. Thus, not achieving this minimum return represents a loss or a drawdown.





Figure 1 Figure 1 (below) displays the net asset value of these three investment strategies:

Figure 1: NAVs of the "Long", "Sorted" and "Flattened" Strategies from 2000 to 2016 (Source: Bloomberg)

These three strategies offer identical annualized return, volatility, downside-volatility, Omega ratio, and even the same VaR and CVaR figures. Thus, without seeing the NAV chart and only viewing the classic risk-return metrics, an investor would be indifferent: all three strategies would look equally good. An investor would be at risk of choosing the Sorted Strategy which exhibits much bigger drawdowns than the Flattened or Long Strategy (hidden risk).

The Sharpe Ratio, although a good measure of adjusted performance, clearly should not be the sole investment criterion. To obtain a more accurate and actionable view of the true global risk of an investment, investors need to consider the risk of long and/or acute drawdowns.

Maximum drawdown is a measure of the maximum decline of the NAV from its historical peak (Calmar Ratio being its return-adjusted version). Although it is a path dependent metric, maximum drawdown is not representative of the global risk of an investment because it only focuses on the worst observed drawdown and not on the length and recurrence of each drawdown.

The section that follows presents a risk measure developed by Martin (1987) to take drawdown risk into consideration.



Ulcer Index (UI): Consider Drawdowns

The Ulcer Index ("Ul," or "Ulcer") measures both the depth and duration of drawdowns and is one of the rare risk indicators that is path-dependent. Its return-adjusted version is a better indicator than the Sharpe Ratio for investors who are more concerned by drawdowns (vs. volatility). The name of the index comes from the supposition that drawdowns cause stress and ulcers to investors.

Ulcer is calculated by taking the quadratic mean⁴ of the drawdowns⁵:

$$UI = \sqrt{\frac{1}{T} \sum_{t=1}^{T} DD_t^2} ,$$

where $DD_t = \frac{P_t}{\max_{i \in [0,t]} P_i} - 1$ is the drawdown at date *t*.

The reader can note a similarity with the definition of standard deviation (where the deviation of returns is replaced by the drawdowns):

$$SD = \sqrt{\frac{1}{T} \sum_{t=1}^{T} (r_t - \bar{r})^2} \quad ,$$

Therefore, Ulcer only takes into account the "downside" risk of a strategy (drawdown equals 0 when the price is at its maximum value).

Table 1

| | Flattened Strategy | Long Strategy | Sorted Strategy |
|-----------------------|--------------------|---------------|-----------------|
| Annualized Return | 1.75% | 1.75% | 1.75% |
| Annualized Volatility | 15.5% | 15.5% | 15.5% |
| Ulcer Index | 3.7% | 24.4% | 86.4% |

Table 1: Annualized returns, volatility and UI of the three strategies (Source: Bloomberg)

Indeed, for the same returns and volatilities, the Ulcer of the Flattened Strategy is nearly seven times smaller than the Ulcer of the Long Strategy and twenty-three times smaller than the Ulcer of the Sorted Strategy. Were an investor to consider Ulcer as their principal risk measure, they would have a persuasive reason for choosing the Flattened Strategy.

⁴ This measure is a convex risk metric as shown in Chekhlov et al. (2005) and annex 1.

⁵ A simplified Ulcer Index (sometimes called the Pain Index) can be defined by taking the arithmetic mean.



Martin (1987) also defines the Ulcer Performance Index (UPI) which is the return divided by Ulcer. UPI is the Ulcer counterpart of the Sharpe Ratio.

Just like volatility, Ulcer represents an average risk and does not take into consideration tail risk. To create a global measure of risk, the next part of this paper will study the tail events of drawdowns.

To summarize our argument thus far:

Average Risk = Ulcer Index

Conditional Drawdown at Risk (CDaR)

As shown in the previous section, Martin was able to define a risk measure (UPI) somewhat comparable to a Sharpe Ratio by changing the data of reference in the calculation from returns to drawdowns. In a similar fashion, Chekhlov et al. (2005)⁶ define DaR and CDaR for Drawdown at Risk and Conditional Drawdown at Risk, comparable to the VaR and CVaR definition.

We define DaB and CDaB as the VaB and CVaB of the Drawdowns distribution (and not the return distribution). Therefore, DaR(α) represents the 1- α % biggest drawdowns and CDaR(α) is the mean of the drawdowns strictly under the $DaR(\alpha)$.

The DaR(95%) and CDaR(95%) of the S&P 500 can be observed in Figure 2:



Figure 2: Cumulative Distribution of Drawdowns, DaR and CDaR definitions of the SP500 from 1990 to 2016 (Source: Bloomberg)

6 Chekhlov et al. (2005) show that DaR and CDaR measures are convex and therefore offer the possibility of convex optimization to create an efficient frontier.



Figure 3 Figure 3 shows the visual interpretation of the CDaR(95%) on the underwater curve of the S&P 500.



Figure 3: CDaR(95%) for the SP500 from Jan. 1990 to Mar. 2016 (Source: Bloomberg)

The CDaR corresponds to the average of the drawdowns falling under the black line which is the DaR(95%) = 43.3%. In this example, CDaR(95%) = 48.2%.

We assume that an investor's level of pain varies on each drawdown. The sharper and/or the longer the drawdown, the higher the discomfort. The most important factor is what investors are ready to stomach based on the risk they believe they are taking. When investing in a strategy with 1% vs. 10% volatility, one does not expect the same drawdown. For this reason, we measure CDaR in multiples of volatility to create a new risk metric: The Pitfall Indicator.

Pitfall Indicator (PI): Measuring the Surprises

In the following paragraph, we create an indicator to measure the bad surprises an investor can expect⁷. An investor is sort of "trapped" inside a drawdown, and can be surprised by an abnormal depth, hence the name: Pitfall Indicator ("PI" or "Pitfall").

The Pitfall Indicator is defined as:

$$PI(\alpha) = \frac{CDaR(\alpha)}{Vol}$$

⁷ This oxymoron has been made on purpose.



The Pitfall represents the average loss of the biggest drawdowns expressed in units of volatility. Thus, the bigger the number, the riskier is the strategy. The Pitfall also addresses one of the main flaws of Ulcer explained below. Let us consider the following two stereotyped strategies:



Figure 4

Figure 4: Net Asset Value of 2 stereotyped strategies (log-scale)

| | Steady Strategy | Unpredictable Strategy |
|-------------------|-----------------|------------------------|
| Annualized Return | 7.5% | 7.5% |
| Volatility | 6.0% | 6.0% |
| Ulcer Index | 7.0% | 7.0% |
| CDaR(95%) | 10% | 40% |
| Pitfall Indicator | 1.64 | 6.31 |

Table 2

Table 2: Statistics of the 2 stereotyped strategies

Both strategies offer the same returns, volatility, and Ulcer, but it is clear that they do not offer the same drawdown profiles. The Steady Strategy exhibits frequent but small drawdowns resulting in a Pitfall of 1.64 (times the volatility of the strategy). The Steady Strategy presents a low risk of surprises to an investor.



On the other hand, the Unpredictable Strategy exhibits infrequent but deep drawdowns resulting in a Pitfall of 6.31 (times its volatility). The Unpredictable Strategy causes a big problem as no rational investor would expect a drawdown over 40% at this level of volatility (6%). This unexpected sharp drawdown will most likely result in the investor cutting his losses and not waiting for a potential recovery.

The Pitfall indicator reflects the number of volatility an investor risks losing by being trapped in a drawdown and should not be ignored in the context of portfolio construction. Considering the following two assets:

- A strategy with a volatility of 5% and a CDaR of 10% (Pitfall=2)
- A strategy with a volatility of 20% and a CDaR of 20% (Pitfall=1)

An investor using the classic risk parity (1/Vol) methodology would allocate 4 times more money to the first strategy. In doing so, the investor greatly increases the risk of a blow-up since the strategy is more likely to reach a deeper drawdown vis-à-vis the volatility (as reflected in the value of the Pitfall).

A strategy can exhibit low average risk (that is, low Ulcer Index or volatility), yet suffer rare extreme drawdowns. It makes sense to penalize such strategies as their average risk does not represent the actual risk. Therefore, we argue that:

Extreme Risk Penalty = Pitfall Indicator = Vol



The Serenity Ratio (SR)

In this section, we create the Serenity Ratio, a Sharpe Ratio equivalent using a penalized risk measure instead of volatility.

First, we use the Pitfall to penalize our previously defined measure of average risk:

Penalized Risk = Average Risk x Extreme Risk Penalty

Penalized Risk = Ulcer x Pitfall

This measure of Penalized Risk represents the average risk of a strategy (Ulcer) adjusted by a measure of extreme risk (Pitfall). Thus, the new measure of risk considers all three-metrics defined previously in this paper: Volatility, Ulcer and CDaR.

By definition:

- The lower the Ulcer the lower the risk of drawdowns
- The lower the Pitfall the lower the risk of surprises

Therefore, just like volatility, the lower the Penalized Risk, the less risky the strategy.

Serenity Ratio ("SR," or "Serenity"), is defined as Return divided by Penalized Risk:

Serenity Ratio = Return Penalized Risk

The name is derived from the fact that the higher the value of Serenity, the more serene an investor will feel in regards to his investment decision. The Serenity is comparable to the Sharpe Ratio, the higher its value the better is the strategy.

Illustration with Hedge Fund Strategies

In the following, we use: The Hedge Fund Research Indices⁸ (HFRI) as proxies for various hedge funds strategies; the S&P 500 and the Barclays Capital Aggregate Bond Index to model investments in equity and bonds⁹ over the 1990 to 2016 time period. The purpose of this section is to show the added-value of the alternative risk measures in the context of a portfolio construction process using various hedge-fund strategies.

⁸ List of indexes used is provided in annex 2.

⁹ All indexes are taken total return (dividend reinvested).



| | Normal Distri- bution | S&P 500 | Barclays US Bond Index | Equity Hedge | Equity Market Neutral | Event Driven | FI Convert. Arb. | Global Macro | Multi- Strategy | Equity Quant. Dir. | Relative Value | Syst. Diversi- fied |
|--------------------------|-----------------------------|------------|------------------------------|-----------------|-----------------------------|-----------------|------------------------|-----------------|--------------------|--------------------------|-------------------|---------------------------|
| Sparkline (Log-Scale) | | M | | m | | | | | | - Martin | | |
| Return | 5.9% | 5.9% | 2.7% | 7.9% | 3.0% | 7.0% | 4.6% | 7.0% | 4.4% | 7.4% | 5.9% | 6.0% |
| Volatility | 14.4% | 14.4% | 3.6% | 8.7% | 2.9% | 6.6% | 6.3% | 7.1% | 4.2% | 12.1% | 4.2% | 7.3% |
| Sharpe | 0.41 | 0.41 | 0.75 | 0.91 | 1.02 | 1.07 | 0.73 | 0.98 | 1.06 | 0.61 | 1.41 | 0.82 |
| Rank | | 11 | 8 | 6 | 4 | 2 | 9 | 5 | 3 | 10 | 1 | 7 |
| Ulcer Index | 18% | 20.2% | 2.6% | 8.1% | 3.7% | 6.0% | 6.1% | 4.4% | 5.1% | 12.5% | 3.4% | 4.2% |
| Ulcer Perf Ind. | 0.33 | 0.29 | 1.05 | 0.98 | 0.80 | 1.18 | 0.75 | 1.59 | 0.87 | 0.59 | | 1.41 |
| Rank | | 11 | 5 | 6 | 8 | 4 | 9 | 2 | 7 | 10 | 1 | 3 |
| CDaR(95%) | -37% | -48.2% | -7.1% | -24.2% | -10.2% | -20.5% | -22.2% | -10.3% | -18.0% | -31.7% | -13.3% | -10.5% |
| Pitfall Ind. | 2.55 | 3.34 | | 2.78 | 3.51 | 3.12 | 3.53 | 1.45 | 4.30 | 2.62 | 3.17 | 1.44 |
| Rank | | 8 | 3 | 5 | 9 | 6 | 10 | 2 | 11 | 4 | 7 | 1 |
| Penalized Risk | 45.0% | 67.7% | 5.1% | 22.5% | 12.9% | 18.6% | 21.6% | 6.3% | 22.0% | 32.6% | 10.9% | 6.1% |
| Serenity | 0.13 | 0.09 | 0.54 | 0.35 | 0.23 | 0.38 | 0.21 | 1.10 | 0.20 | 0.23 | | 0.98 |
| Rank | | 11 | 4 | 6 | 7 | 5 | 9 | 1 | 10 | 8 | 3 | 2 |
| 11 We | orst | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 Bes | st |

Table 3

Table 3: Advanced Risk Metrics for different Hedge Fund Strategies for the 1990-2016 period (Source: Bloomberg)

The Sharpe Ratio and Ulcer Performance Index in Table 3 exhibit the weakness - indeed, the danger — of not properly acknowledging the "extreme" risk of an investment.

Relative Value strategies present the best risk adjusted metrics through Sharpe Ratio and Ulcer Performance Index. However, for the Pitfall Indicator they fall to 7th place with extreme drawdowns reaching 3.17 times their level of volatility. This could prove harmful to an investor in the context of a classic risk parity allocation as the investor would have allocated more capital towards those strategies looking solely at volatility. These strategies display steady returns but their actual drawdowns can be sharper than expected. Investors choosing to allocate to Relative Value strategies run the risk of having their long-term steady performance gains wiped out by a rare and deep drawdown.

While strategies such as Global Macro and Systematic Diversified seem less appealing when considering their Sharpe Ratio (5th and 7th place respectively), they are amongst the top three strategies in terms of Ulcer Performance Index. They also outclass all other strategies when using the Pitfall Indicator. While providing less steady returns, Systematic Diversified and Global Marco Strategies are less likely to incur heavy drawdowns compared to their level of volatility. These strategies have less hidden risk.



Comparing classic and alternative risk metrics on hedge-fund strategies exacerbates the necessity of managing both average and extreme risk during the allocation process. Using only the Sharpe Ratio (hence considering volatility as the only measure of risk) can lead to allocating more capital towards strategies with a higher probability of blow-up. This risk can be mitigated through the consideration of both the Ulcer Index and the Pitfall Indicator through the Serenity Ratio.

An Alternative Portfolio Theory

The Modern Portfolio Theory (MPT) developed by Markowitz (1952) uses volatility as the sole risk metric. The allocation process proposed by Markowitz is driven by attaining the maximum Sharpe Ratio on the Portfolio.

We propose to adapt MPT by maximizing the Serenity Ratio. In the classic Risk-Return space, we replace Volatility with our Penalized Risk Metric.

Figure 5 displays the values of Returns and Penalized Risk for every strategy. The lines drawn on the graph represent indifference lines for the investor as the value of the Serenity Ratio stays constant over each line. A rational investor always prefers strategies in the upper left quadrant to strategies in the bottom right quadrant, as they present a higher measure of Serenity. Investors would therefore be better off investing in Systematic Diversified or Global Macro hedge funds if they wish to maximize their performance relative the risk of drawdowns looking at Ulcer and Pitfall.

The proximity between Sharpe and Serenity will be further developed in our second white paper by introducing the definition of an efficient frontier analogous to the theory developed by Markowitz (1952) and previously studied in Harris and Mazibas (2011), Allen et al. (2015) and Goldberg and Mahmoud (2015).





Figure 5



Figure 5: Serenity Ratios for the 1990-2016 period (Source: Bloomberg)



Conclusion

Investors should be aware of the incomplete nature of using classic risk metrics such as volatility, Sharpe Ratio, VaR, and CVaR as inputs into investment decisions. All these measures share the same flaw: Considering time-independence of returns (and leaving out any potential auto-correlation). Thus, we believe the use of Ulcer and Pitfall in addition to these widely used risk measures is advised if an investor wishes to have a better understanding of both known and hidden risks of a strategy. Indeed, when comparing many hedge fund strategies using only classic risk measures, we find that some strategies seem very appealing when investors only use volatility as a risk measure, therefore maximizing the Sharpe Ratio. However, the perception of global risk of these strategies is better depicted by including the risk of drawdowns, through Ulcer and especially, through Pitfall. Replacing the classic Markowitz representation (Mean-Variance) with both the Ulcer and the Pitfall (Mean-Penalized Risk) introduces a new and interesting manner of sorting strategies. The use of these alternative risk metrics reduces the risk of an investment blow-up by preventing misallocation to riskier strategies. This Alternative Portfolio Theory will be further developed in a second white paper, taking into account the correlation of drawdowns and defining an alternative efficient frontier



References and Appendices



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Appendices – Part 1

Annex 1 – Coherent Risk Measure

A coherent risk measure ρ as defined by Artzner & al. (1999) must respect the following properties:

(1) Normalized

 $\rho(0) = 0$

The risk of holding no assets is zero.

(2) Monotonicity

If X_1 and X_2 are two portfolios and $X_1 \leq X_2$ almost surely, then $\rho(X_1) \geq \rho(X_2)$.

The risk of a better portfolio is always lower.

(3) Positive Homogeneity

If $\lambda > 0$ then $\rho(\lambda X) = \lambda \rho(X)$

Which means that if you double your portfolio you double your risk.

(4) Insensitivity to a constant shift

If a is a constant, $\rho(X + a) = \rho(X)$

(5) Sub-additivity

If X_1 and X_2 are two portfolios

$\rho(X_1+X_2)<\rho(X_1)+\rho(X_2)$

The risk of two portfolios together cannot be worse than adding the risks of each portfolio. This is the concept of diversification.

(6) Convexity

The notion of sub-additivity and positive homogeneity can be replaced by the notion of convexity:

for
$$\lambda \in [0,1] \rho(\lambda X_1 + (1 - \lambda) X_2) < \lambda \rho(X_1) + (1 - \lambda)\rho(X_2)$$

Ulcer Index

(1), (2), (3), (4) are a direct consequence of the definition.

(6) Convexity: even though the Ulcer Index is defined as a norm in the mathematical sense and therefore is convex we will show it here.

$$UI(X) = \sqrt{\frac{1}{T}\sum_{i=0}^{T} x_i^2} = \sqrt{\sum_{i=0}^{T} y_i^2} = ||Y||_2 \text{ where } Y = \frac{X}{\sqrt{T}} \text{ and } ||.||_2 \text{ is the euclidean norm}$$



Using (3), we only have to show that the Euclidean norm is convex. Therefore, we have to prove that its unit ball is convex.

We have:

$$\begin{aligned} \forall \lambda \in [0,1] \,\forall X, Y \in B_2 \ \parallel \lambda X + (1-\lambda)Y \parallel_2^2 &= \sum_{i=0}^T (\lambda x_i + (1-\lambda)y_i)^2 \\ &\leq \sum_{i=0}^T \lambda x_i^2 + (1-\lambda)y_i^2 \text{ because } f \colon x \mapsto x^2 \text{ is convex } (f''(x) = 2 > 0) \\ &= \lambda \parallel X \parallel_2^2 + (1-\lambda) \parallel Y \parallel_2^2 \leq 1 \end{aligned}$$

Therefore $\lambda X + (1 - \lambda)Y \in B_2$ which means B_2 is convex and the norm is also convex.

Thus the Ulcer Index is a convex risk measure.

Annex 2 – HFRI Indices

| HFRI Names | Short Name | Strategies Included | | | |
|--|--|---|--|--|--|
| HFRI Macro: Systematic Diversified (HFRIMTI Index) | Systematic Diversified | Managed Futures, Trend Index Following | | | |
| HFRI EH: Equity Market Neutral Index (HFRIEMNI Index) | Equity Market Neutral | Quantitaztive Equity Market Neutral Strategies | | | |
| HFRI EH: Quantitative Directional (HFRIENHI Index) | Equity Quantitative Directional | Factor-Based and Statistical Arbitrage Trading Strategies | | | |
| HFRI RV: Fixed Income-Convertible Arbitrage Index (HFRICAI Index) | Fixed Income- Convertible Arbitrage | Relative Value Strategies limited to Fixed Income and Convertible Instruments | | | |
| HFRI RV: Multi-Strategy Index (HFRIFI Index) | Multi-Strategy | Relative Value Strategies on Fixed Income, derivatives, Equity, Real Estate and/or MLP Assets | | | |
| HFRI Event-Driven (Total) Index (HFRIEDI Index) | Event-Driven | Event Driven Strategies | | | |
| HFRI Equity Hedge (Total) Index (HFRIEHI Index) | Equity Hedge | Long-Short Equity Strategies | | | |
| HFRI Macro (Total) Index (HFRIMI Index) | Global Macro | Global Macro Strategies | | | |
| HFRI Relative Value (Total) Index (HFRIRVA Index) | Relative Value | Relative Value Strategies | | | |
| S&P 500 (SPXT Index) | S&P 500 | S&P 500 | | | |
| Barclays US Bond Index (LBUSTRUU Index) | Barclays US Bond Index | US Bonds | | | |



Table 4

Annex 3 – Autocorrelation of Hedge Fund Indexes

By definition, auto-correlation is defined as the correlation between the values of the same process at different times. We have tested autocorrelation of lag 1 for all the indexes of annex 2, in order to detect indexes where returns are somewhat linked to previous returns. Indeed, one can see that if an asset exhibits high autocorrelation, it means that when negative returns occur, they persist over time and cause drawdowns. Therefore, for strategies with high autocorrelation, it is not abnormal to find deep drawdowns and therefore high pitfall indexes.

The table below presents the AR(1) (Lag 1 Auto regressive coefficient) based on monthly data from 1990 to 2016 and their statistical significance according to the Ljung-Box test (* represent statistical significance at the 5% level).

| | Syst. Diversi- fied | Equity Market Neutral | Equity Quant. Dir. | Convert. Arb. | Multi- Strategy | Event Driven | Equity Hedge | Global Macro | Relative Value | S&P 500 | Barclays US Bond Index |
|-------|---------------------------|-----------------------------|--------------------------|------------------|--------------------|-----------------|-----------------|-----------------|-------------------|------------|------------------------------|
| AR(1) | 3.5% | 7.0% | 19.6%* | 57.2%* | 50.4%* | 37.6%* | 22.0%* | 14.8%* | 43.8%* | 5.3% | 12.4% |

Table 4: Lag 1 Autocorrelation (* represents significance at the 5% confidence level)





Figure 6

20% 10% 0% -10%

Systematic

Diversified

Equity

Market

Neutral

Figure 6: Lag 1 Autocorrelation for the 1990-2016 period (blue color represents significance at the 5% confidence level)

Quant.

Dir.

Equity Fixed Income Multi-

Arbitrage

Convert. Strategy

We can see that many hedge-fund strategies exhibit statistically significant autocorrelation through the 1990-2016 period. Fixed-Income convertible arbitrage and Multi-strategy hedge funds even exhibit coefficient above 0.5 showing strong autocorrelation of returns. As a conclusion, it is statistically sound to find that the returns of some strategies tend to persist over time creating period of inevitable drawdowns.

Event

Driven

Equity

Hedge

Global

Macro

Relative

Value

S&P

500

Barclays

US Bond

Index



Notes

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